5 bodů

(1) $\int x \cdot \operatorname{arctg}(x)\,dx = \left| \begin{array}{ll} f'(x) = x & \to f(x) = \frac{1}{2}x^2 \\ g(x) = \operatorname{arctg}(x) & \to g'(x) = \frac{1}{1+x^2} \end{array} \right| =$

$D_f = \mathbb{R} \Rightarrow$ PF na $\mathbb{R}$

$= \frac{1}{2}x^2 \operatorname{arctg}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = \frac{1}{2}x^2\operatorname{arctg}(x) - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx =$

$= \frac{1}{2}x^2\operatorname{arctg}(x) - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)dx = \frac{1}{2}x^2\operatorname{arctg}(x) - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg}(x) =$

$= \underline{\frac{1}{2}\left[(x^2+1)\operatorname{arctg}(x) - x\right] \text{ na } \mathbb{R}}$

5 bodů

(2) $\int \cos^5 x\,dx = \left| \begin{array}{l} \text{lichý integrand} \\ \text{v cosinu} \end{array} \right| = \left| \begin{array}{l} t = \sin x\ ;\ x \in \mathbb{R},\ t \in \langle -1;1 \rangle \\ dt = \cos x\,dx \\ \cos^2 x = 1 - \sin^2 x = 1 - t^2 \end{array} \right| =$

$D_f = \mathbb{R} \Rightarrow$ PF na $\mathbb{R}$

$= \int \cos^4 x \cdot \cos x\,dx = \int (1-t^2)^2\,dt = \int (1 - 2t^2 + t^4)\,dt =$

$= t - 2\cdot\frac{t^3}{3} + \frac{t^5}{5} = \underline{\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x \text{ na } \mathbb{R}}$

10 bodů

(3) $\int \frac{x+2}{x^2+2x+3} = \left| \begin{array}{l} 4 - 4\cdot 1\cdot 3 = -8 < 0 \\ \Rightarrow \text{zlomek je již ve tvaru} \\ \text{parciálního rozkladu} \end{array} \right| = \frac{1}{2}\int \frac{2x+4}{x^2+2x+3}\,dx =$

$D_f = \mathbb{R} \Rightarrow$ PF na $\mathbb{R}$

$= \frac{1}{2}\int \frac{2x+2}{x^2+2x+3}\,dx + \frac{1}{2}\cdot 2\int \frac{dx}{x^2+2x+3} = \frac{1}{2}\int \frac{dt}{t} + \int \frac{dx}{(x+1)^2+2} =$

$\left| \begin{array}{l} t = x^2+2x+3 \\ dt = (2x+2)\,dx \end{array} \right| \qquad \left| \begin{array}{l} x^2+2x+3 = x^2 + 2\cdot 1\cdot x + 1 - 1 + 3 = \\ = (x+1)^2 + 2 = 2\left[\left(\frac{x+1}{\sqrt{2}}\right)^2 + 1\right] \end{array} \right|$

$= \frac{1}{2}\ln(x^2+2x+3) + \frac{1}{2}\int \frac{dx}{\left(\frac{x+1}{\sqrt{2}}\right)^2 + 1} = \frac{1}{2}\ln(x^2+2x+3) + \frac{1}{2}\int \frac{\sqrt{2}\,dy}{y^2+1} =$

$\left| \begin{array}{l} y = \frac{x+1}{\sqrt{2}} \\ dy = \frac{1}{\sqrt{2}}\,dx \end{array} \right|$

$= \underline{\frac{1}{2}\ln(x^2+2x+3) + \frac{\sqrt{2}}{2}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) \text{ na } \mathbb{R}}$