1. písemka 7. 3. 2018
ZS5

4b

① $\int \mathrm{cotg}^2(x)\,dx = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{1-\sin^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int dx =$

$D_f = \mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$

$= \ldots\mathrm{cotg}(x) - x \quad \text{na} \quad \mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$

6b

② $\int \frac{\sin x \cos^3 x}{1+\cos^2 x}\,dx = \left| \begin{array}{l} \underbrace{\cos^2 x}_{\in\langle 0;1\rangle} = t \\ -2\cos x \sin x\,dx = dt \end{array} \right| = -\frac{1}{2} \int \frac{(-2\cos x \sin x)\cos^2 x\,dx}{1+\cos^2 x} =$

$D_f = \mathbb{R}$

$= -\frac{1}{2} \int \frac{t\,dt}{1+t} = -\frac{1}{2} \int \frac{(t+1-1)\,dt}{1+t} = -\frac{1}{2} \int 1\cdot dt + \frac{1}{2} \int \frac{dt}{1+t} =$

$= -\frac{1}{2} t + \frac{1}{2} \ln|1+t| \underset{t\in\langle 0;1\rangle}{=} -\frac{1}{2} t + \frac{1}{2} \ln(1+t) = -\frac{1}{2}\cos^2 x + \frac{1}{2}\ln(1+\cos^2 x)$ na $\mathbb{R}$

10b

③ $\lim_{x\to\frac{\pi}{2}} \frac{\mathrm{tg}(x)}{\mathrm{tg}(3x)} = \left| \begin{array}{l} \text{předp. l'H:} \\ 1.\ \lim_{x\to\frac{\pi}{2}} |\mathrm{tg}(3x)| = +\infty \\ 2.\ \checkmark,\ 3.\ \exists \to \text{uvidíme} \end{array} \right| \overset{l'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2(3x)}\cdot 3} =$

$= \frac{1}{3} \lim_{x\to\frac{\pi}{2}} \frac{\cos^2(3x)}{\cos^2(x)} = \left| \text{"}\frac{0}{0}\text{"} \right| \overset{l'H}{=} \cancel{\frac{1}{3}} \lim_{x\to\frac{\pi}{2}} \frac{\cancel{-2}\cos(3x)\cdot\sin(3x)\cdot\cancel{3}}{\cancel{-2}\cos(x)\cdot\sin(x)} =$

$= \lim_{x\to\frac{\pi}{2}} \frac{\cos(3x)\cdot\sin(3x)}{\cos(x)\cdot\sin(x)} = \left| \begin{array}{l} \sin(\frac{\pi}{2}) = 1 \\ \sin(\frac{3}{2}\pi) = -1 \end{array} \right| \overset{VOAL}{=} -\lim_{x\to\frac{\pi}{2}} \frac{\cos(3x)}{\cos(x)} = \left| \text{"}\frac{0}{0}\text{"} \right| \overset{l'H}{=}$

$= -\lim_{x\to\frac{\pi}{2}} \frac{\cancel{-}\sin(3x)\cdot 3}{\cancel{-}\sin(x)} = -3 \cdot \frac{\overbrace{\sin\left(\frac{3\pi}{2}\right)}^{=-1}}{\underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}} = (-3)\cdot(-1) = \underline{\underline{3}}$